

# Traffic Flow Dynamics

Martin Treiber · Arne Kesting

# Traffic Flow Dynamics

Data, Models and Simulation

Translated by Martin Treiber and Christian Thiemann

 Springer

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# Preface

In order to keep people moving in times of rising traffic and limited resources, science is challenged to find intelligent solutions. Over the past few years, contributions from engineers, physicists, mathematicians, and behavioral psychologists have led to a better understanding of driver behavior and vehicular traffic flow. This interdisciplinary field will surely produce further advances in the future. The focus is on new applications ranging from novel driver-assistance systems, to intelligent approaches to optimizing traffic flow, to the precise detection of traffic jams and the short-term forecasting of traffic for dynamic navigation aids.

This textbook offers a comprehensive and didactic account of the different aspects of vehicular traffic flow dynamics and how to describe and simulate them with mathematical models. We hope to make this fascinating field accessible to a broader readership; to date, it has only been documented in specialized scientific papers and monographs.

Part I describes how to obtain and interpret traffic flow data, the basis of any quantitative modeling. The second and main part is devoted to the different approaches and models used to mathematically describe traffic flow. The starting point of most models are the basic concepts of physics—many-particle systems, hydrodynamics, and classical Newtonian mechanics—augmented by behavioral aspects and traffic rules. At the website<sup>1</sup> accompanying this book, the reader can interactively run a selection of traffic models and reproduce some of the simulation results displayed in the figures. Part III gives an overview of major applications including traffic-state estimation, fuel consumption, and emission modeling, determining travel times (the basis of dynamic navigation), and how to optimize traffic flow.

The book is written for students, lecturers, and professionals of engineering and transportation sciences and for interested students in general. It also offers material for project work in programming, numerical methods, simulation, and mathematical modeling at college and university level. The reference implementations in the

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<sup>1</sup> see: [www.traffic-flow-dynamics.org](http://www.traffic-flow-dynamics.org)

multi-model open-source vehicular traffic simulator *MovSim*<sup>2</sup> can be used as a starting point for the reader's own simulation experiments and model development.

This work originates from the lecture notes of courses in traffic flow dynamics and modeling at the Dresden University of Technology, Germany; these have been previously published, by the same publisher, in the German book “Verkehrsdynamik und Simulation”. The English edition has been updated and significantly extended to include new topics, e.g., on model calibration. To underline its textbook character, it contains many problems with elaborated solutions.

We thank all colleagues at our Department for Traffic Econometrics and Modeling at the Dresden University of Technology, particularly Dirk Helbing, for various scientific discussions and stimulations. We would also like to thank Marietta Seifert, Christian Thiemann, and Stefan Lämmer for suggestions and corrections. Special thanks go to Martin Budden for reviewing the manuscript as a native English speaker. He is also one of the main contributors to *MovSim*. Finally, we would like to thank Martina Seifert, Christine and Hanskarl Treiber, Ingrid, Bernd, and Dörte Kesting, Claudia Perlitius, and Ralph Germ who contributed to the book with valuable suggestions.

Dresden, June 2012

Martin Treiber  
Arne Kesting

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<sup>2</sup> see: [www.movsim.org](http://www.movsim.org)

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